حل واجبات مسائل الاهتزازات (4)

السؤال الأول :

$$[K] = \begin{bmatrix} k_1 + k_2 & -k_2 \\ k_2 & k_2 \end{bmatrix} = 10^6 \begin{bmatrix} 120 & -60 \\ 60 & 60 \end{bmatrix}$$

$$[M] = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} = 10^3 \begin{bmatrix} 6 & 0 \\ 0 & 12 \end{bmatrix}$$

$$\left| [K] - w^2 [M] \right| = 0$$

$$\left| \begin{bmatrix} 12\times10^6 & -6\times10^6 \\ -6\times10^6 & 6\times10^6 \end{bmatrix} - w^2 \begin{bmatrix} 6\times10^3 & 0 \\ 0 & 12\times10^3 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 12\times10^6 - 6\times10^3 w^2 & -6\times10^6 \\ -6\times10^6 & 6\times10^6 - 12\times10^3 w^2 \end{bmatrix} \right| = 0$$

$$\left[ (12\times10^6 - 6\times10^3 w^2)(6\times10^6 - 12\times10^3 w^2) - (6\times10^6)(6\times10^6) \right] = 0$$

$$= 12\times10^6 \times 6 \times 10^6 - 12\times10^6 \times 12 \times 10^3 W^2 -$$

$$- 6\times10^3 w^2 \times 6 \times 10^6 + 6\times10^3 w^2 \times 12 \times 10^3 w^2 - 6\times10^6 \times 6\times10^6$$

$$= 3,6\times10^{13} - 1,8\times10^{11} w^2 + 72\times10^6 w^4$$

$$= 72\times10^6 w^4 - 1,8\times10^{11} w^2 + 3,6\times10^{13} = 0$$

$\lambda = \omega^2$

$$72\lambda^2 - 1{,}8 \times 10^5 \times 3{,}6 \times 10^7 = 0$$

$$\omega_1^2 = 219{,}22 \Rightarrow \omega_1 = 14{,}8 \text{ rad/Sec}$$

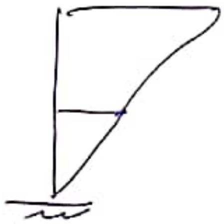
$$\omega_2^2 = 2280{,}77 \Rightarrow \omega_2 = 47{,}76 \text{ rad/Sec}$$

ايجاد اشكال التشوه mode shapes

① 

$$\begin{bmatrix} 10{,}68 \times 10^6 & -6 \times 10^6 \\ -6 \times 10^6 & 3{,}37 \times 10^6 \end{bmatrix} \times \begin{bmatrix} y_{11} \\ y_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} y_{11} \\ y_{12} \end{bmatrix} = \begin{bmatrix} 1 \\ 1{,}78 \end{bmatrix}$$



② 

$$\begin{bmatrix} -1{,}685 \times 10^6 & -6 \times 10^6 \\ -6 \times 10^6 & -21{,}37 \times 10^6 \end{bmatrix} \begin{bmatrix} y_{21} \\ y_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} y_{21} \\ y_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ -0{,}281 \end{bmatrix}$$

مصفوفة اشكال $[\phi]$

$$[\phi] = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$$

②

$$\phi_{11} = \frac{y_{11}}{\sqrt{m_1 y_{11}^2 + m_2 y_{21}^2}} = 4{,}76 \times 10^{-3}$$

$$\phi_{21} = \frac{y_{21}}{\sqrt{m_1 y_{11}^2 + m_2 y_{21}^2}} = 8{,}483 \times 10^{-3}$$

$$\phi_{12} = 0{,}012 \qquad \phi_{22} = -3{,}371 \times 10^{-3}$$

$$\phi = \begin{bmatrix} 4{,}766 \times 10^{-3} & 0{,}012 \\ 8{,}483 \times 10^{-3} & -3{,}371 \times 10^{-3} \end{bmatrix}$$

$$[P(t)] = [\phi]^T \{Pf\}$$

$$\begin{bmatrix} P_1(t) \\ P_2(t) \end{bmatrix} = \begin{bmatrix} 1192{,}41 \\ 776{,}1 \end{bmatrix} N$$

$$\left.\begin{aligned} \ddot{z}_1 + \omega_1^2 z_1 &= P_1(t) \\ \ddot{z}_2 + \omega_2^2 z_2 &= P_2(t) \end{aligned}\right\} \quad z(t) = z_{st}(1 - \cos\omega t)$$

$$z_{st_1} = 5{,}44 \Rightarrow z_{1max} = 198$$

$$z_{2max} = 0{,}677$$

$$\begin{Bmatrix} y_{1max} \\ y_{2max} \end{Bmatrix} = [\phi] \begin{bmatrix} z_{1max} \\ z_{2max} \end{bmatrix} = \begin{bmatrix} 0{,}0597 \\ 0{,}089 \end{bmatrix}$$

ـ الثاني :

$$M_e = \int_0^L \bar{m}\,[\phi]^2\,dx + \sum_{i=1}^{n} m\,[\phi x]^2$$

مفترض

$$\phi(x) = \left(1 - \cos\frac{\pi x}{2L}\right)$$

$$\phi'(x) = \frac{\pi}{2L}\cos\frac{\pi x}{2L}$$

$$\phi''(x) = \frac{\pi^2}{4L^2}\cos\frac{\pi x}{2L}$$

$$M_e = \int_0^{21} 400\left[1 - \cos\frac{\pi x}{2\times 21}\right]^2 dx + 3\times 10^4\left(1 - \cos\frac{\pi x}{2\times 21}\right)^2$$

$$M_e = 31904{,}859$$

$$K_e = \int_0^L EI\left[\frac{\pi^2}{4L^2}\cdot\cos\frac{\pi x}{2L}\right]^2 dx$$

$$K_e = 220{,}878\times 10^8\ N/m$$

$$\omega = \sqrt{\frac{K_e}{M_e}} = 83{,}2\ rad/sec$$

$$T = 0{,}0755\ sec$$

$$y(t) = y_{st}\,(1 - \cos\omega t)$$

$$F_e(t) = \int_0^L P(t)\,\phi(x)\,dx$$

$$F_e(t) = \int_0^{21} -400\times 0.71\times 9{,}81\left(1 - \cos\frac{\pi x}{2L}\right)$$

$-3\times10^{-4}\left[1-\cos\frac{\pi x}{2L}\right]\times 0,35\times 9,81$

$F_e(t) = -113,5\times10^{3}\ N$

$y_{st} = \frac{f_0}{k_e} = -5,138\times10^{-4}$

$y(t) = -5,138\times10^{-4}(1-\cos 83,2t)$

$y_{max} = 1,026\times10^{-3}\ m$

$V_{max} = k_e\, y_{max} = 226,62\ kN$